Lecture no.8

Transport Properties of Nonideal Plasmas

Introduction

It is known that electrophysical properties of plasma are primarily defined by the electron component. Electrical conductivity weakly nonideal plasma at $\Gamma << 1$ can be determined by well known Spitzer theory. For strongly coupled plasma ($\Gamma > 1$) we use a computer simulation molecular dynamic method. The transport properties of nonideal plasma at the moderate values of coupling parameter can be investigated by kinetic equation methods.

Electrical conductivity of weakly ionized plasma

The electrical conductivity σ is defined by the number density of electrons n_e and their mobility μ:

$$\sigma = e\mu n_e \tag{1}$$

In the case of nonideal plasma, these quantities are connected by well known expressions from kinetic theory. The number densities of electrons n_e and ions n_i are related by the Saha formula (see, lecture no.5):

$$\begin{cases} n_i \cdot n_e = K_1 n_a \,; \\ K_1 = \dfrac{2\sum_i}{\sum} \left(\dfrac{m k_B T}{2\pi\hbar^2} \right)^{3/2} \cdot \exp(-I / k_B T) \end{cases}, \tag{2}$$

where n_a is the number density of atoms; $\sum$ and $\sum_i$ are statistical sums of atoms and ions, respectively; I is the ionization potential. Due to the absence of complex ions in an ideal plasma, we have $n_i = n_e$. At low temperatures the degree of ionization is low ($n_e << n_a$):

$$n_e = \sqrt{K_1 n_a} \, . \tag{3}$$

Electron–ion and electron–electron interactions can be ignored in a weakly ionized plasma, therefore, we consider only interactions of electrons with atoms (molecules) of a neutral gas. Such model of plasma is called the Lorentz gas model.

We will use the Boltzmann equation in order to derive the expression for mobility of electrons. In stationary and spatially homogeneous cases the Boltzmann kinetic equation for the distribution function of electrons $f(\upsilon)$ in an electric field $\vec{F}$ has the following form:

$$-\left(\frac{e\vec{F}}{m}\right)\partial f(\vec{\upsilon})/\partial\vec{\upsilon} = I_c(f) . \quad (4)$$

It should be noted that the left–hand side of this equation describes the field effect and the right–hand side is responsible for the variation of the number of electrons in an element of phase volume due to collisions; $I_c(f)$ is the collision integral. We assume small deviations of $f(\upsilon)$ from equilibrium due to the fact that the electron mass is much smaller than the atomic mass. Then, $f(\upsilon)$ should be close to spherically symmetric and can be represented as follows:

$$f(\vec{\upsilon}) = f_0(\vec{\upsilon}) + \cos\vartheta\, f_1(\vec{\upsilon}) , \quad (5)$$

where ϑ is the angle between the directions of the velocity and electric field. Under conditions of thermodynamic equilibrium the symmetric part of the distribution function $f_0(\upsilon)$ is maxwellian. Notice that the nonsymmetric part $f_1(\upsilon)$ is important for calculation the electron mobility, and, consequently for investigation of plasma electrical conductivity.

Substituting the expression (5) for $f(\upsilon)$ in the kinetic equation (4), we obtain the following formula:

$$-\left(\frac{e\vec{F}}{m}\right)\partial f_0(\vec{\upsilon})/\partial\vec{\upsilon} = I_c(f_1) . \quad (6)$$

The direction of electron's velocity is strongly changed at each collisions and this direction does not depend on their velocities before collisions. Then

$$I_c(f_1) = -\nu(\vec{\upsilon})\, f_1(\vec{\upsilon})\,;\quad \nu(\vec{\upsilon}) = n_a \upsilon q(\vec{\upsilon}) \quad , \qquad (7)$$

where $q(\vec{\upsilon})$ is the transport cross section of electron–atom scattering and $\nu(\vec{\upsilon})$ is the corresponding electron–atom collision frequency. The electrons are mainly in chaotic thermal motion and drift in the direction opposite to the field $\vec{F}$. The drift velocity ($\vec{\omega} = -\mu\vec{F}$) is defined as the mean electron velocity over the time exceeding greatly the time between individual collisions and given by the following expression:

$$\omega = \int \upsilon \cos\vartheta f(\vec{\upsilon}) d\vec{\upsilon} = \int \upsilon \cos^2\vartheta f_1(\vec{\upsilon}) d\vec{\upsilon} \quad , \qquad (8)$$

because $f_0(\upsilon)$ does not make contribution to ω. Substituting the expression for $f_1(\upsilon)$ from kinetic equation in the formula for drift velocity ω, we have the following relation for mobility of electrons $\mu = \omega / F$:

$$\mu = \frac{e}{m} \int \frac{\partial f_0}{\partial \vec{\upsilon}} \cos^2\vartheta \frac{\vec{\upsilon} d\vec{\upsilon}}{\nu(\vec{\upsilon})} \quad . \qquad (9)$$

Integrating over the angles and substituting the maxwellian distribution for $f_0(\upsilon)$, and using the fact that $\partial f_0 / \partial \vec{\upsilon} = -f_0 \vec{\upsilon} / \upsilon_T^2$ we finally get:

$$\mu = \frac{1}{3}\sqrt{\frac{2}{\pi}} \frac{e}{m\upsilon_T^5} \int_0^\infty \frac{\upsilon^4}{\nu(\vec{\upsilon})} \exp\left(-\frac{\upsilon^2}{2\upsilon_T^2}\right) d\upsilon \quad , \qquad (10)$$

here $\upsilon_T = \sqrt{k_B T / m}$ is the thermal velocity of electrons. Expressions (9) and (10) describe the mobility of electrons in the Lorentz plasma approximation. In should be noted that formula (10) is useful for calculation of electron's mobility in the real plasma. But in this case we should know electron-atom collision frequency $\nu(\upsilon)$.

The expressions (9) and (10) are valid at the following conditions:

1) The binary collision approximation is valid and the neutral gas must be sufficiently rarefied, i.e. $n_a q^{3/2} << 1$.
2) The temperature must be sufficiently high and the thermal wavelength of the electron sufficiently small, so that we can ignore the interference of the electron on atoms, i.e. $n_a q \lambda_e << 1$.
3) The potential energy of the Coulomb interaction between electrons must be much smaller than their kinetic energy, i.e. $e^2 n_e^{1/3} / k_B T << 1$.
4) The plasma must be nondegenerate, i.e. $\hbar^2 n_e^{2/3} / m k_B T << 1$.
5) The correlation between atoms can be neglected, i.e. $n_a b << 1; \quad n_a a / k_B T << 1$ (a and b are the coefficients of the van der Waals equation of state for the neutral gas).

It is convenient to integrate over the electron energy $E = m\upsilon^2 / 2$ instead of the velocity. Then, we have for mobility the following expression:

$$\mu = \frac{4}{3}\sqrt{\frac{1}{\pi}}\frac{e}{m(k_B T)^{5/2}}\int_0^\infty \frac{E^{3/2}}{\nu(E)}\exp\left(-\frac{E}{k_B T}\right)dE \quad , \qquad (11)$$

where the collision frequency is $\nu(E) = n_a q(E)\sqrt{2E/m}$. Introducing the mean (effective) collision frequency $\bar{\nu}$ and cross section $\bar{q}$, one can write:

$$\begin{cases} \mu = \dfrac{e}{m\bar{\nu}}; \\ \bar{\nu} = \left(3\sqrt{2\pi}/4\right) n_a \bar{q}(T)\upsilon_T \end{cases} \quad , \qquad (12)$$

where

$$\frac{1}{\bar{q}(T)} = \frac{1}{(k_B T)^2}\int_0^\infty \frac{E}{q(E)}\exp\left(-\frac{E}{k_B T}\right)dE \qquad (13)$$

In the simplest case when the electron–neutral collision can be approximated as a scattering on a hard sphere of a diameter d, the transport cross section is independent of energy, i.e. $q(E) = \pi d^2/4$. In this case by averaging over energies we have the following values:

$$\begin{cases} \bar{q} = \pi d^2/4; \\ \bar{\nu} = (3\pi^{3/2}/2^{7/2}) n_a d^2 \upsilon_T \end{cases} . \qquad (14)$$

In the case of real plasma the transport cross section is a function of energy. One can conclude that if the dependence $q(E)$ is known, the mean cross sections $\bar{q}(T)$ can also be easily calculated. A large amount of reference data on electron–atom and electron–molecule scattering cross–sections is available from special books. As an example the data of averaged cross–sections for atoms of alkali metals are shown in the table 1.

T, 10^3 K	Li	Na	K	Cs	T, 10^3 K	Li	Na	K	Cs
1.0	16.5	15.0	15.3	14.1	2.6	6.88	7.31	7.00	8.98
1.2	14.4	14.0	13.6	12.8	2.8	6.41	6.73	6.52	7.63
1.4	12.6	12.9	12.1	11.7	3.0	5.99	6.23	6.10	7.32
1.6	11.1	11.7	10.9	10.8	3.2	5.63	5.79	5.73	7.04
1.8	9.91	10.6	9.84	10.1	3.4	5.32	5.41	5.41	6.79
2.0	8.93	8.73	8.96	9.42	3.6	5.05	5.07	5.12	6.57
2.2	8.11	8.73	8.20	8.89	3.8	4.80	4.77	4.87	6.37
2.4	7.46	7.97	7.56	8.41	4.0	4.59	4.51	4.64	6.20

Table 1. Averaged transport cross sections of scattering of electrons from atoms of alkali metals, $\bar{q}(T)$ in units of $10^2 a_0^2$.

The electrical conductivity of plasma can be estimated by the following expression:

$$\sigma \cong 3.8 \cdot 10^6 \frac{n_e}{n_a} \frac{1}{\bar{q}\sqrt{T}} \; ohm^{-1} cm^{-1} , \qquad (15)$$

where $\bar{q}$ is the average cross section in units of $10^{-16}\,cm^2$ and T is the temperature in K.